Enrollment No: _____   Exam Seat No:_____

C.U.SHAH UNIVERSITY
Winter Examination-2018

Subject Name : Advanced Mathematics

Subject Code : 2TE02AMT1   **Branch: Diploma (All)**

Semester : 2   **Date : 23/10/2018**   **Time : 02:30 To 05:30**   **Marks : 70**

Instructions:
  (1) Use of Programmable calculator & any other electronic instrument is prohibited.
  (2) Instructions written on main answer book are strictly to be obeyed.
  (3) Draw neat diagrams and figures (if necessary) at right places.
  (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**

a) The distance between the points $(1, 3)$ and $(0, -4)$ is _____.
   (A) 50   (B) $5\sqrt{2}$ (C) $2\sqrt{5}$ (D) None of these

b) If $(3, 8)$, $(4, 2)$ and $(-1, 5)$ are the vertices of a triangle, then the co ordinates of its centroid is _____
   (A) $(2, -5)$ (B) $(-2, 5)$ (C) $(2, 5)$ (D) None of these

c) Slope of the line $2x - 3y + 4 = 0$ is _____.
   (A) $-2/3$ (B) $-3/2$ (C) $3/2$ (D) $2/3$

d) Radius of the circle $x^2 + y^2 = 7$ is _____ .
   (A) 7   (B) $\sqrt{7}$ (C) $\dfrac{7}{2}$ (D) None of these

e) $\lim\limits_{x \to \infty} \left(1 + \dfrac{1}{x}\right)^x = $ _____
   (A) e^2 (B) e (C) $e^{1/2}$ (D) None of these

f) $\lim\limits_{x \to \infty} x\left[\sqrt[x]{7} - 1\right] = $ _____
   (A) 0   (B) e^7 (C) $\log_e 7$ (D) 1

g) $\lim\limits_{x \to 1} \dfrac{x^2 + 3x + 4}{x + 2} = $ _____
   (A) 8/3   (B) 3/8   (C) 2   (D) 3

h) $\dfrac{d\left(a^x\right)}{dx} = $ _____
   (A) $a^x \log_e a$ (B) $\log_e a$ (C) $x^a \log_e a$ (D) $a^x \log_e x$

i) $\dfrac{d\left(x^2 + 2x + 7\right)}{dx} = $ _____
   (A) $2x$ (B) $2x + 1$ (C) $2x + 2$ (D) None of these

j) $\dfrac{d(\log\cos x)}{dx} = $ _____

(A) $\cot x$ (B) $-\tan x$ (C) $\cos ecx$ (D) $\sec x$

k) $\dfrac{d\left(\tan^{-1}x + \cot^{-1}x\right)}{dx} = $ _____

(A) -1 (B) 0 (C) 1 (D) None of these

l) $\displaystyle\int \sec^2 x\, dx = $ _____

(A) $\tan x + c$ (B) $\cot x + c$ (C) $\sec x \tan x + c$ (D) $-\cos ecx \cot x + c$

m) $\displaystyle\int \dfrac{1}{\sqrt{x^2 + a^2}}\, dx = $ _____

(A) $\log\left|x + \sqrt{x^2 + a^2}\right| + c$ (B) $\log\left|x + \sqrt{x^2 - a^2}\right| + c$ (C) $\dfrac{1}{2a}\log\left|\dfrac{x-a}{x+a}\right| + c$

(D) $\dfrac{1}{2a}\log\left|\dfrac{a+x}{a-x}\right| + c$

n) $\displaystyle\int_0^1 \dfrac{2}{1+x^2}\, dx = $ _____

(A) π (B) $\dfrac{\pi}{4}$ (C) $\dfrac{\pi}{2}$ (D) None of these

Attempt any four questions from Q-2 to Q-8

Q-2   **Attempt all questions**   **(14)**

a) In which ratio Y – axis divides line segment joining points (1, 2) and (2, 1)? Find   **(5)**
co ordinates of division point.

b) Find the equation of straight line passing through (3, 4) and parallel to line   **(5)**
$\dfrac{x}{2} + \dfrac{y}{2} = 1$.

c) If $f(x) = \dfrac{1-x}{1+x}$ then prove that $f(x) + f\left(\dfrac{1}{x}\right) = 0$.   **(4)**

Q-3   **Attempt all questions**   **(14)**

a) Prove that $\displaystyle\lim_{x\to 3} \dfrac{\sqrt{x+2} - \sqrt{5}}{\sqrt{x+4} - \sqrt{7}} = \dfrac{\sqrt{35}}{5}$.   **(5)**

b) Evaluate: $\displaystyle\lim_{\theta\to 0} \dfrac{\cos ec\theta - \cot\theta}{\theta}$ **(5)**

c) A(– 1, 3), B(– 1, x) and C(4, 3) are vertices of a triangle. If $m\angle B = 90°$ then find value   **(4)**
of x.

Q-4   **Attempt all questions**   **(14)**

a) Find centre and radius of the circle $36x^2 + 36y^2 + 24x - 36y - 23 = 0$.   **(5)**

b) Find derivative of $f(x) = e^x$ using definition.   **(5)**

c) Find $\dfrac{dy}{dx}$ if $y = e^x \sin x\, \cos x$.   **(4)**

Q-5   **Attempt all questions**   **(14)**

a) Evaluate: $\displaystyle\lim_{n\to\infty} 4\left[\dfrac{1^3 + 2^3 + 3^3 + \ldots + n^3}{n^4}\right]$ **(5)**

b) The equation of motion of a particle is $S = 2t^3 + 3t^2 - 12t + 5$. **(5)**
(i) Find velocity at $t = 0$. (ii) Find acceleration at $t = 1$.

c) Find $\dfrac{dy}{dx}$ if $y = (\sin x)^{\tan x}$. **(4)**

Q-6 **Attempt all questions** **(14)**

a) Evaluate: $\displaystyle\int \dfrac{e^x(1+x)}{\cos^2(xe^x)} dx$ **(5)**

b) Find $\dfrac{dy}{dx}$ if $ax^2 + by^2 + 2hxy + 2gx + 2fy + c = 0$. **(5)**

c) Evaluate: $\displaystyle\int \dfrac{(1-3x)^2}{x^3} dx$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Find $\dfrac{dy}{dx}$ if $y = \log\left[x + \sqrt{x^2 + a^2}\right]$. **(5)**

b) If $\dfrac{dy}{dx} = 4x^2 + 6x - 1$ and $y = 5$ when $x = 2$, represent y as a function of x. **(5)**

c) Evaluate: $\displaystyle\int x\, e^x\, dx$ **(4)**

Q-8 **Attempt all questions** **(14)**

a) Prove that $\displaystyle\int_0^{\frac{\pi}{2}} \dfrac{\sec x}{\sec x + \cos ecx} dx = \dfrac{\pi}{4}$. **(5)**

b) Find the area of the circle $x^2 + y^2 = r^2$. **(5)**

c) Find the equation of a circle passing through point $(-7, 1)$ and centre $(-4, -3)$. **(4)**